THE GENESIS OF FUZZY SETS AND SYSTEMS - ASPECTS IN SCIENCE AND PHILOSOPHY

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Contents

Summary

In 1965 Lotfi A. Zadeh founded the theory of Fuzzy Sets and Systems. This chapter deals with developments in the history of philosophy, logic, and mathematics during the time before and up to the beginning of fuzzy logic and it also gives a view of its first application in control theory. Regarding the term ''fuzzy'' we note that older concepts of ''vagueness'' and ''haziness'' had previously been discussed in philosophy, logic, mathematics. This chapter delineates some specific paths through the history of the use of these ''loose concepts''. Haziness and fuzziness were concepts of interest in mathematics and philosophy during the second half of the 20th century. The logico-philosophical history presented here covers the work of Russell, Black, Hertz, Wittgenstein and others. The mathematical–technical history deals with the theories founded by Menger and Zadeh. Menger's concepts of probabilistic metrics, hazy sets (ensembles flous) and micro-geometry as well as Zadeh's theory of Fuzzy Sets paved the way for the establishment of Soft Computing methods.

In the first decade of Fuzzy Sets and Systems, nobody thought that this theory would be successful in the field of applied sciences and technology. Zadeh expected that his theory would have a role in the future of computer systems as well as Humanities and Social Sciences. When Mamdani and Assilian picked up the idea of Fuzzy Algorithms to establish a first Fuzzy Control system for a small steam engine, this was the Kick-off for the "Fuzzy-Boom" and Zadeh's primary intention trailed away for years. Then in the new millennium a new movement for Fuzzy Sets in Social Sciences and Humanities was launched.

1. Introduction

In the summer of 2015 we will commemorate the 50th anniversary of the theory of Fuzzy Sets and Systems (FSS). This is a fuzzy jubilee because the exact time when Lotfi A. Zadeh (born 1921) discovered the concept of fuzzy sets is unknown. The event is roughly assigned to the summer of 1965.

This chapter reviews the genesis of this new mathematical theory following my original (historical) research work that encompasses inspections of scientific articles, newspapers, letters, and – most importantly – interviews with Lotfi A. Zadeh and other protagonists of the early years of the theory of FSS. For details see Seising R. (2007).

In Sections 2 and 3 we briefly follow Zadeh's development as an electrical engineer in the 1950s and 1960s from concrete Network theory and Filter theory to abstract System theory and then to the theory of Fuzzy sets and Systems as a generalized System theory. In Section 4, we present short views on historical paths of modern logic and philosophy of mathematics in the 20th century, the logical analysis of vagueness, the concepts of *statistical metrics* and *ensembles flous* and we consider Wittgenstein's late philosophy in relation to our subject. Section 5 gives a review of Zadeh's works on Fuzzy Sets in language and meaning that appeared before the "Fuzzy Boom" with real-world application systems that is the subject of Section 6. Section 7 gives an outlook on Zadeh's later theories: *Computing with words* and *with perceptions*.

2. From Electrical Engineering to System Theory

Fuzzy Sets and Systems can look back upon an eventful story in the scientific environment of electrical engineering, including the initial system theory and computer sciences known during this time, which were part of Zadeh's training as a student in Tehran, Iran. Following his immigration to the USA in 1942, Zadeh continued his studies at the Massachusetts Institute of Technology (MIT) in Cambridge, Massachusetts. He moved to New York in 1946, where he was awarded a Ph.D by Columbia University in 1949. Since 1958, he has been a Professor of Electrical Engineering at the University of California at Berkeley. When he established the theory of fuzzy sets in the mid-1960s, he was already a well-known protagonist of the system theoretical approach in electrical engineering, which was a new scientific trend from the 1950s onward. Together with Charles A. Desoer (1926-2011) he published in 1962 *Linear System Theory: The State Space Approach* (Zadeh and Desoer, 1963), which became a standard textbook. In 1963, together with Elijah Polak he edited the volume *System Theory* (Zadeh and Polak,1969). In his own contribution to this volume, which was entitled "The Concepts of System, Aggregate, and State in System Theory" (Zadeh,1969), Zadeh presented his state space approach. Two years later, when he introduced fuzzy sets, he construed his new theory as a "general system theory".

Figure 1. Lotfi A. Zadeh, Charles A. Desoer, and Elijah Polak:
colleagues at the University of California, Berkeley.

In 1954 – Zadeh was then an instructor at Columbia University in New York — he wrote for the *Columbia Engineering Quarterly* an article, named "System Theory" (Zadeh, 1954) that begins as follows:
"If you never heard of system theory, you need not feel like an ignoramus. It is not one of the well-established branches of science. In fact, it has not yet been officially recognized as a scientific discipline. It does not appear on programmes of meetings of scientific societies nor in indices to scientific publications. It does not have well-defined boundaries, nor does it have settled objectives." (Zadeh, 1954, p. 34). Zadeh emphasized that all scientific disciplines are concerned with systems, but the new branch, named system theory, considers systems as mathematical constructs rather than physical objects: "The distinguishing characteristic of system theory is its abstractness." (Zadeh, 1954, p. 16) In this and later papers Zadeh quoted the definition of a "system" from *Webster's Dictionary*: A system is "an aggregation or assemblage of objects united by some form of interaction or interdependence." (Figure 2)

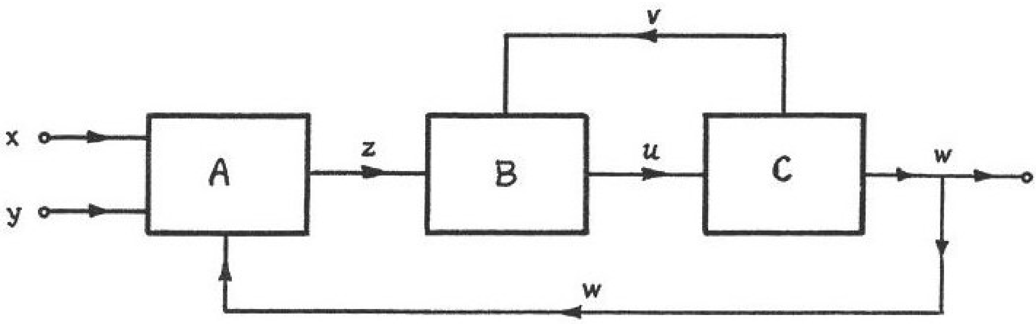

Figure 2. Block diagram of interconnected objects, (Zadeh,1969, p. 17).

System theorists deal with abstract systems, "that is, systems whose elements have no particular physical identity" (Zadeh,1969, p. 16); they deal with "black boxes". Figure 3 reproduces the illustration to this article – actually a "black box"!

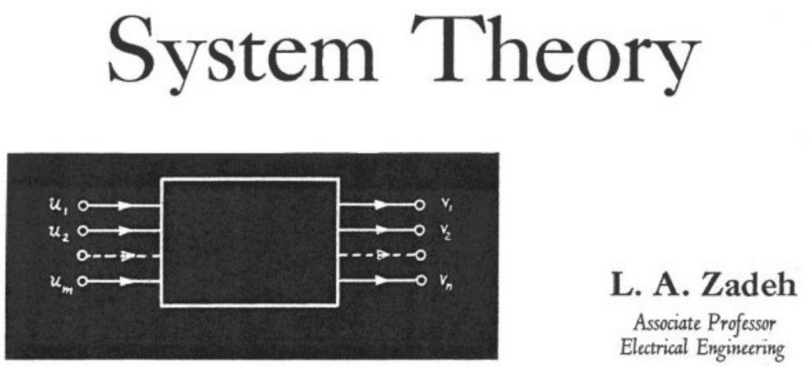

Figure 3. Illustration from Zadeh's article (Zadeh,1969).

Communication systems are a special type of systems that have been of interest since the 1950s, when information and communication theory emerged as successful scientific and technological disciplines. Zadeh was deeply involved in the development of this new communication theory and its techniques when he delivered a lecture on "Some Basic Problems in Communication of Information" at the meeting of the Section of Mathematics and Engineering of the New York Academy of Sciences in March 1952 (Zadeh, 1952). He represented signals as ordered pairs ($x(t), y(t)$)of points in a signal space Σ, which is imbedded in a function space with a delta-function basis (Figure 6). This analogy between projection in a function space and filtration by an ideal filter led Zadeh to postulate a function symbolism of filters in the early 1950s [7]. Thus, $N = N_1 + N_2$ represents a filter consisting of two filters connected by addition, $N = N_1 N_2$ represents their tandem (sequential) combination and $N = N_1 | N_2$ the separation process (Figure 4).

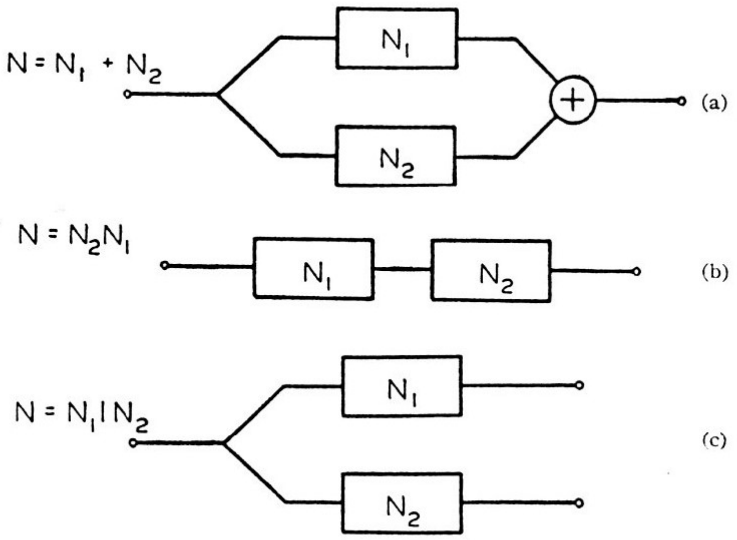

Figure 4. Functional symbolism of ideal filters, (Zadeh and Miller, 1952, p. 225).

Later (in (Zadeh, 1953), Zadeh discussed the concept of optimal filters as opposed to ideal filters following Wiener's work. Ideal filters are defined as filters that achieve a perfect separation of signal and noise. However, in reality there are no ideal filters, e.g., an ideal low-pass filter should retain all frequency components until a certain threshold until which all components should be fully suppressed. In practice, we cannot have such a step-shaped separation. Zadeh knew that we get a smooth transition from 1 to 0 transmission coefficient as the frequency decreases. These transitions are similar to the well known membership functions of fuzzy sets. However, in the 1950s the time was not ripe for this new mathematical theory. Zadeh defined optimal filters as those that give the "best approximation" of a signal, and he noted that "best approximations" depend on reasonable criteria. At this time he formulated these criteria in statistical terms.

But starting in the next decade he wrote the landmark article "From Circuit Theory to System" for the anniversary edition of the *Proceedings of the IRE* that appeared in May 1962 to mark the 50th year of the *Institute of Radio Engineers* (IRE) (Zadeh, 1962). Here, he could outline problems and applications of system theory and its relations to network theory, control theory, and information theory. Furthermore, he pointed out "that the same abstract 'systems' notions are operating in various guises in many unrelated fields of science is a relatively recent development. It has been brought about, largely within the past two decades, by the great progress in our understanding of the behaviour of both inanimate and animate systems—progress which resulted on the one hand from a vast expansion in the scientific and technological activities directed toward the development of highly complex systems for such purposes as automatic control, pattern recognition, data-processing, communication, and machine computation, and, on the other hand, by attempts at quantitative analyses of the extremely complex animate and man-machine systems which are encountered in biology, neurophysiology, econometrics, operations research and other fields" (Zadeh, 1962, p. 856f.).

In this article Zadeh used for the very first time the word "fuzzy" and he wrote it down to characterize his vision of new mathematics: "In fact, there is a fairly wide gap between what might be regarded as "animate" system theorists and "inanimate" system theorists at the present time, and it is not at all certain that this gap will be narrowed, much less closed, in the near future. There are some who feel that this gap reflects the fundamental inadequacy of the conventional mathematics – the mathematics of precisely-defined points, functions, sets, probability measures, etc. – for coping with the analysis of biological systems, and that to deal effectively with such systems, which are generally orders of magnitude more complex than man-made systems, we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions." (Zadeh, 1962, p. 857). However, when Zadeh published these notions, he did not know what this mathematics of fuzzy quantities would look like.

Another method to deal with imperfect or noisy signals in communication systems was introduced in the 1950s by Richard E. Bellman (1920-1984), a young mathematician working at the RAND Corporation, United States Air Force Project in Santa Monica, California. Bellman was the founder of the method of *Dynamic Programming* (Bellman, 1957), and tried to apply his "principle of optimality" in communication theory.

In the late 1950s, Bellman met Zadeh in New York, where Zadeh worked at Columbia University. Their friendship lasted until Bellman's death in 1984. Even though they considered diverse mathematical aspects of electrical engineering, system theory and, later, computer science, they met each other very often and discussed several aspects of their scientific work. Bellman was the first and most important critic of Zadeh's new theory of fuzzy sets in 1965.

Figure 5. Richard E. Bellman, Robert E. Kalaba, and Lotfi A. Zadeh.

3. A New View on System Theory: Fuzzy Sets and Systems

Zadeh and Bellman planned to work together at RAND in Santa Monica, California, in the summer of 1964. Prior to the summer of 1964, Zadeh gave a talk on pattern recognition at the *Wright-Patterson Air Force Base*, Dayton, Ohio. It may have been on this occasion that he started thinking about the use of grades of membership for pattern classification and that he conceived the first example of fuzzy mathematics, which he wrote in one of his first papers on the subject: "For example, suppose that we are concerned with devising a test for differentiating between handwritten letters *O* and *D*. One approach to this problem would be to give a set of handwritten letters and indicate their grades of membership in the fuzzy sets *O* and *D*. On performing abstraction on these samples, one obtains the estimates / and / of μ_O and μ_D, respectively. Then given a letter *x* which is not one of the given samples, one can calculate its grades of membership in *O* and *D*; and, if *O* and *D* have no overlap, classify x in *O* or *D*." (Zadeh, 1965a, p. 30)

In a few days he extended this concept to the theory of fuzzy sets and a few weeks later, he discussed this preliminary version of the theory of fuzzy sets with Bellman. Then he wrote his manuscript on "Fuzzy Sets" (Zadeh, 1965b) and submitted it to the journal *Information and Control* in November 1964. "Fuzzy Sets" appeared in June 1965 and was the first article on fuzzy sets in a scientific journal. However, Lotfi Zadeh also wrote other papers at the time. According to common practice at the department of electrical engineering in Berkeley, the article "Fuzzy Sets" was preprinted as a report of the *Electronics Research Laboratory* in November 1964 (Zadeh, 1964). As a result of his talk in Dayton, Ohio, he wrote a paper which he sent to Bellman who was the editor of the *Journal of Mathematical Analysis and Applications*. Bellman agreed to publish the paper in said journal but the publication appeared late, in 1966, under the title "Abstraction and Pattern Classification" (Bellman and Kalaba and Zadeh 1966). The authors of the article were Bellman, his associate Robert E. Kalaba and Zadeh (see Figure 5). The text and the authors' names are identical to those of the RAND memorandum RM-4307-PR, which appeared as early as October 1964. This memo was written by Zadeh alone. Here he defined fuzzy sets for the first time in a scientific paper, establishing a general framework for the treatment of pattern recognition problems (Bellman and Kalaba and Zadeh 1964).

In April 1965 the Symposium on System Theory was held at Polytechnic Institute in Brooklyn. At this meeting Zadeh presented "A New View on System Theory": a view that deals with the concepts of fuzzy sets, "which provide a way of treating fuzziness in a quantitative manner." In the subsequent publication of the proceedings of this symposium we find a shortened manuscript version of the talk. His contribution was entitled "Fuzzy Sets and Systems" in this publication: (Zadeh, 1965a, p. 29). In this lecture and in the paper, Zadeh first defined "fuzzy systems" as follows:

> A system S is a *fuzzy system* if (input) $u(t)$, output $y(t)$, or state $s(t)$ of S or any combination of them ranges over fuzzy sets, (Zadeh, 1965a, p. 33).

Zadeh explained that "these concepts relate to situations in which the source of imprecision is not a random variable or a stochastic process but rather a class or classes which do not possess sharply defined boundaries." (Zadeh, 1965a, p. 29) His "simple" examples in this brief summary of his new "way of dealing with classes in which there may be intermediate grades of membership" were "the "class" of real numbers which are much larger than, say, 10, and "the "class" of "bald man", and also the "class" of adaptive systems. For further details on the roots of fuzzy systems in system theory, the reader is referred to the author's book (Zadeh, 1965a).

In "Fuzzy Sets" (Zadeh, 1965b), Zadeh introduced the new mathematical entities "fuzzy sets": "Such classes are not classes or sets in the usual sense of these terms, since they do not dichotomize all objects into those that belong to the class and those that do not." He introduced "the concept of a fuzzy set, that is a class in which there may be a continuous infinity of grades of membership, with the grade of membership of an object x in a fuzzy set A represented by a number $\mu_A(x)$ in the interval [0,1]." Zadeh maintained that these new concepts provide a "convenient way of defining *abstraction* — a process which plays a basic role in human thinking and communication." (Zadeh, 1965a, p. 29) The question was how to generalize various concepts, union of sets, intersection of sets, and so forth. Zadeh defined *equality*, *containment*, *complementation*, *intersection* and *union* relating to fuzzy sets A, B in any universe of discourse X as follows (for all $x \in X$):

- $A = B$ if and only if $\mu_A(x) = \mu_B(x)$, (1)
- $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$, (2)
- $\neg A$ is the complement of A if and only if $\mu_{\neg A}(x) = 1 - \mu_A(x)$, (3)
- $A \cup B$ if and only if $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$, (4)

- $A \cap B$ if and only if $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$. (5)

For his interpretation of fuzzy unions and intersections he wrote a separate paragraph, which shows a very important analogy to sieves, because Zadeh wrote: "Specifically, let $f_i(x), i = 1, \ldots, n$, denote the value of the membership function of A_i at x. Associate with $f_i(x)$ a sieve $S_i(x)$ whose meshes are of size $f_i(x)$. Then, $f_i(x) \vee f_j(x)$ and $f_i(x) \wedge f_j(x)$ correspond, respectively, to parallel and series combinations of $S_i(x)$ and $S_j(x)$," as shown in Figure 6.

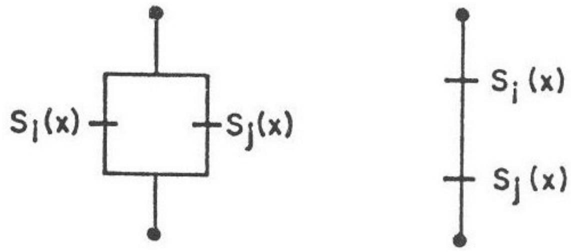

Figure 6. Parallel and serial combination of sieves illustrating the fuzzy union, $\cup$, (maximum) and intersection, $\cap$, (minimum), [12].

More generally, a well-formed expression involving $A_1, \ldots, A_n, \cup$ and $\cap$ corresponds to a network of sieves $S_1(x), \ldots S_n(x)$ which can be found by the conventional synthesis techniques for switching circuits. As a very simple example,

$$C = [(A_1 \cup A_2) \cap A_3] \cup A_4 \quad (6)$$

corresponds to the network shown in Figure 7." ([12], p. 334-344.)

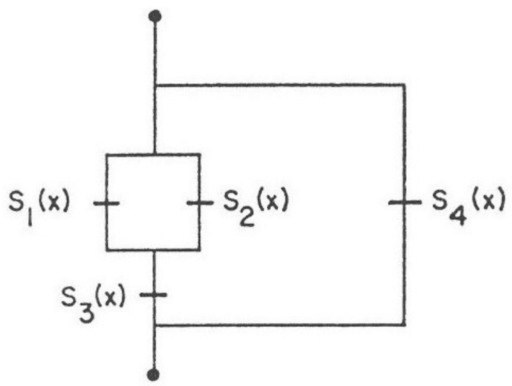

Figure 7. A network of sieves simulating $\{(f_1(x) \vee f_2(x)) \wedge f_3(x)) \vee f_4(x)\}$, Zadeh, 1965b.

If the reader takes into account the fact that the term "sieve" denotes a filter, he will comprehend the analogy of fuzzy sets and electrical filters as outlined in the first section.

In the decade that followed the first publications on Fuzzy Sets and Systems (Zadeh, 1965a, Zadeh1965b, Zadeh 1964, Zadeh, 1966, Bellman und Kalaba and Zadeh 1964) Zadeh expected that they would have a role in the future of computer systems as well as humanities and social sciences. At that time nobody thought that this theory would be successful in the field of applied sciences and technology. Quite the contrary, he remarked that he did not expect the incorporation of FSS into the fields of sciences and engineering: “What we still lack, and lack rather acutely, are methods for dealing with systems which are too complex or too ill-defined to admit of precise analysis. Such systems pervade life sciences, social sciences, philosophy, economics, psychology and many other “soft” fields.” (Zadeh, 1971)

After “Fuzzy Sets” had appeared in print, Zadeh received many requests for offprints. Philosopher Max Black who had published a paper entitled “Vagueness – An Exercise in Logical Analysis” back in 1937 anticipated a vague idea from Zadeh’s theory (Black, 1937), for he wrote: “The vagueness of the word chair is typical of all terms whose application involves the use of the senses. In all such cases,‛ ‛borderline case’ and ‛doubtful objects’ are easily found to which we are unable to say either that the class name does or does not apply.” (Black, 1937), p. 434) He now told Zadeh in a letter at June21, 1967: “You were good enough to send me, some time ago, some of your recent papers on topics connected with “Fuzzy Sets”. If I have not written before, the reason has not been lack of interests, but an inescapable press of other duties. Now that I have had a chance, at least, to study your work, I want to express my admiration and interest. I believe that your ingenious construction promises to provide intellectual tools of great value. In case you have not come across it, I might draw your attention to an early article of mine ... “ He referred to his article (Black, 1937), that was also already reprinted in his book, *Language and Philosophy* (Black, 1949) and also to his “more recent article on similar topics ..”: “Reasoning with Loose Concepts” (Black, 1963). To understand the history of the logical analysis of vagueness let’s go back to the history of modern science and its philosophy!

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Menger K. (1951c). Ensembles flous et fonctions aléatoires, *Comptes Rendus Académie des Sciences* **37,** 226--229. [In this paper that was written by Menger when he was a visiting professor in Paris, he introduced the concept of 'ensembles fluos' or 'hazy sets'. Today 'ensembles flous' are the French name for fuzzy sets but Menger's old concept of 'ensembles flous' was still probabilistic and not identical with fuzzy sets.]

Menger K. (1970). Mathematical implications of Mach's ideas: Positivistic geometry, the clarification of functional connections. In: Cohen R.S., Seeger R.J. (eds.). *Ernst Mach, Physicist and Philosopher, Boston Studies in the Philosophy of Science*, vol. 6, Dordrecht: Reidel, 107-125. Reprint as: Menger K. Geometry and Positivism. A Probabilistic Microgeometry. In: Menger K. Selected Papers in Logic and Foundations, Didactics, Economics. Dordrecht: D. Reidel Publ. Comp., 225--234 (1979). Citation after the reprinted version. [This article is the printed form of Menger's contribution to the symposium of the American Association for the Advancement of Science, organized in 1966 to commemorate the 50th anniversary of Ernst Mach's death. In this contribution he compared his "micro geometry" or probabilistic geometry (see (Menger K., 1951a) with the theory of fuzzy sets. However, he mentioned the difference between his concept of hazy sets and Zadeh's concept of fuzzy sets because he wrote that Zadeh spoke "of the degree rather than the probability of an element belonging to a set".]

Rosch, E. (1973). Natural Categories, *Cognitive Psychology* **4**, 328--350. [In this article the psychologist Rosch demonstrated – based on a series of experiments in the 1970s -- that when people label an object, they rely on a comparison with what they regard as a prototype of the category designated by that word but less on abstract definitions.]

Russell B. (1923). Vagueness, *Aust. J. Psychol. Philos.* **1**, pp. 84–92. [In this article the mathematician and philosopher Bertrand Russell discusses some of the most important problems concerning the nature of vagueness, its extension within the language, and its relation to truth and logic.]

Rutherford D.A. (1976). The Implementation and Evaluation of a Fuzzy Control Algorithmus for a Sinter Plant. In: Mamdani E.H., Gaines B.R. (eds.). Discrete Systems and Fuzzy Reasoning, EES-MMS-DSFR-76. Proceedings of Workshop held at Queen Mary College, University of London. [This paper presents an early fuzzy control application system of a sinter making plant in England.]

Seising R, Sanz V. (2012a). Introduction. In: (Seising R, Sanz V. (eds.), 2012b), 3-36 [This introduction to the volume (Seising R, Sanz V. (eds.), 2012b) illuminates the relations between hard and soft sciences and hard and soft computing with each other.]

Seising R, Sanz V. (eds.) (2012b). *Soft Computing in Humanities and Social Sciences*, Berlin, New York, [et al.]: Springer. [This anthology presents a generous sampling of a wide array of authors and subject matters from different disciplines. Some of the contributors of the book belong to the scientific and technical areas of Soft Computing while others come from various fields in the humanities and social sciences such as Philosophy, History, Sociology or Economics.]

Seising R, Sanz V. (guest eds.). (2013) Soft Computing in Humanities and Social Sciences, special issue *Fuzzy Sets and Systems* **214**, 1-96. [This special issue of the journal concerns theoretical research and practical applications of Fuzzy Sets and Systems in non-technical fields.]

Seising R. (2007). *The Fuzzification of Systems. The Genesis of Fuzzy Set Theory and Its Initial Applications–Developments up to the 1970s*. (Studies in Fuzziness and Soft Computing 216) Berlin [et al.]: Springer. [A history of Fuzzy Set Theory and the ways it was first used. The book incorporates this genesis of the new theory into the history of 20th century science and technology. Influences from philosophy, system theory and cybernetics stemming from the earliest part of the 20th century are considered alongside those of communication and control theory from mid-century.]

Seising R. (2012). The Experimenter and the Theoretician -- Linguistic Synthesis to tell Machines what to do. In: Trillas E., Bonissone P., Magdalena L., Kacprycz J. (Eds.). *Combining Experimentation and Theory -- A Homage to Abe Mamdani*, Berlin: Springer, pp. 329-358. [This book contribution concerns the life work of two pioneers of Fuzzy Sets and Systems, Ebrahim H. Mamdani and Lotfi A. Zadeh. Mamdani initiated the development of practical Fuzzy Control systems whereas Zadeh founded the theory of this field. When they have been asked to characterize their own role in science, the former characterized himself less close to mathematics than the other.]

Tong R.M. (1976). An Assessment of a Fuzzy Control Algorithm for a Nonlinear Multivariable system. In: Proc. Workshop on Discrete Systems and fuzzy Reasoning. Queen Mary College, London. [This paper presents an early fuzzy control application system of in a basic oxygen steel making process in England.]

Turakainen P. (1968). On Stochastic Languages, *Information and Control* **12**(4), 304--313. [This article presents a concept of stochastic languages as an approximation to human languages using randomizations in the productions.]

Wee W.G. (1967). On a Generalization of Adaptive Algorithms and Applications of the Fuzzy Set Concept to Pattern Classification. Ph.D Thesis, Purdue University, Tech. Rep. 67(7). [In his Ph D dissertation, written this work under King Sun Fu, We had applied the fuzzy sets to iterative learning procedures for pattern classification and he had defined a finite automaton based on Zadeh's concept of the fuzzy relation as a model for learning systems.]

Wee W.G., Fu, K.S. (1969) A Formulation of Fuzzy Automata and its Application as a Model of Learning Systems. *IEEE Transactions on Systems Science and Cybernetics* **SSC-5**(3), 215--223.

Wittgenstein L. (1914-1916). Werkausgabe in acht Bänden. Frankfurt am Main: Suhrkamp, Bd. 8, 1984. [This is the German edition of Wittgenstein's Collected Works.]

Wittgenstein L. (1921). *Tractatus logico-philosophicus*. Routledge & Kegan Paul, London. (First in German: L. Wittgenstein: Logisch-Philosophische Abhandlung. Ostwalds Annalen der Naturphilosophie, Band 14, Leipzig 1921.) [Wittgenstein's first book concerns the philosophical problems which deal with the world, thought and language. Wittgenstein's "solution" bases in logic and in the nature of representation.]

Wittgenstein L. (1953). *Philosophical Investigations*. Blackwell Publishing, Oxford. [In his second book, published posthumously, Wittgenstein discusses several philosophical problems. On the contrary to his *Tractatus Logico-Philosophicus* he claimed that most philosophical problems root in conceptual confusions surrounding language use.]

Zadeh L. A. (1971b). Quantitative Fuzzy Semantics, *Information Sciences* **3**, 159--176. [In this article Zadeh defined a language as a fuzzy relation between a set of terms $T = \{x\}$ and the universe of discourse $U = \{y\}$. If a term x of T is given, then the membership function $\mu_L(x, y)$ defines a set $M(x)$ in U with the following membership function: $\mu_M x\ (y)\mu_L(x, y)$. Zadeh called the fuzzy set $M(x)$ the *meaning* of the term x; x is thus the *name* of $M(x)$.]

Zadeh L.A. (?). Toward Fuzziness in Computer Systems. Fuzzy Algorithms and Languages. (Script, without date and publisher). [This paper was found by the author in Zadeh's office. The appearance is unknown.]

Zadeh L.A. (1952). Some Basic Problems in Communication of Information, *The New York Academy of Sciences*, March 1952, Series II, **14**(5), 201--204. [Zadeh's talk at the meeting of the Section of Mathematics and Engineering of the New York Academy of Sciences on February 15, 1952.]

Zadeh L.A. (1953). Theory of Filtering, *Journal of the Society for Industrial and Applied Mathematics* **1**, 35--51. [In this article on his general filter theory Zadeh emphasized a distinction between ideal and optimum filters. The former are defined as filters which achieve a perfect separation of signal and noise, but if ideal filtration is not possible, though, which is often the case when the signal is mixed with noise, then one must accept that the filtration can only be incomplete. In such cases, a filter that delivers the best possible approximation of the desired signal – and "a particular meaning" of "best approximation" is used here – is called an optimum filter.]

Zadeh L.A. (1954). System Theory, *Columbia Engineering Quarterly*, November 1954, 16--19, and 34. [Zadeh's first paper on System theory in the New York student publication "Columbia Engineering Quarterly".]

Zadeh L.A. (1962). From Circuit Theory to System Theory, *Proceedings of the IRE* **50**, 856-865. [This article was written for the anniversary edition of the Proceedings of the IRE appeared in May 1962 to mark the 50th year of the Institute of Radio Engineers (IRE). The article presents a brief survey of the evolution of system theory, together with an exposition of some of its main concepts, techniques and problems. It concerns problems and applications of system theory and its relations to network theory, control theory, and information theory. The discussion is centered on the notion of state and emphasizes the role played by state-space techniques.]

Zadeh L.A. (1964). Fuzzy Sets, ERL Report No. 64-44, University of California at Berkeley, November 16, 1964. [Zadeh's preprint of the article [12].]

Zadeh L.A. (1965a). Fuzzy Sets and Systems, in: Fox J. (ed.). *System Theory*. Microwave Research Institute Symposia Series XV, Brooklyn, New York: Polytechnic Press, 29--37. [Zadeh's contributionin the proceedings of the Symposium on System Theory (April 20-22, 1965) at the Polytechnic Institute in Brooklyn, When Zadeh gave this talk it was entitled "A New View on System Theory".]

Zadeh L.A. (1965b) Fuzzy Sets, *Information and Control* **8**, 338--53. [This is the first and seminal article on fuzzy sets.]

Zadeh L.A. (1968) Fuzzy Algorithms, *Information and Control* **12**, 99—102. [In this article Zadeh introduced the concept of fuzzy algorithms.]

Zadeh L.A. (1969). Biological Application of the Theory of Fuzzy Sets and Systems. In: Proctor L.D. (ed.). The Proceedings of an International Symposium on Biocybernetics of the Central Nervous System. London: Little, Brown and Comp., 199-206. [Zadeh's conference paper to advise life scientists to use fuzzy set theory.]

Zadeh L.A. (1969). The concept of state in system theory, in (Zadeh L.A. and Polak E. 1969), 9--42. [This is Zadeh's contribution to (Zadeh L.A. and Polak E. 1969) where he presented the so-called "state space approach" to System Theory.]

Zadeh L.A. (1970). Fuzzy Languages and their Relation to Human and Machine Intelligence. In: Man and Computer. Proc. Int. Conf. Bordeaux 1970, Karger: Basel, 13—165. [In this proceedings paper Zadeh's thesis was that the difference between human and mechanical intelligence lay in the ability of the human brain – "an ability which present-day digital computers do not possess – to think and reason in imprecise, non-quantitative terms". He said that humans could understand inexact instructions, whereas inputs for a computer had to be defined with precision. He suggested devising fuzzy languages which functioned such that commands formulated in a language like this could also be processed and carried out by future computers.]

Zadeh L.A. (1971). Towards a theory of fuzzy systems. In: Kalman R.E., DeClaris N. (eds.). *Aspects of Network and System Theory*, New York: Holt, Rinehart and Winston, 469--490. [Zadeh's contribution to an anthology on Network and System Theory.]

Zadeh L.A. (1971a). Similarity Relations and Fuzzy Orderings. *Information Sciences* **3**, 177--200. [In this article Zadeh defined *similarity relations* as a generalization of the concept of equivalence relations (reflexive symmetrical and transitive) and *fuzzy orderings* as transitive fuzzy relations.]

Zadeh L.A. (1972). A Fuzzy-Set-Theoretic Interpretation of Linguistic Hedges, *J. of Cybernetic*s **2**, 4--34. [Zadeh's article on "linguistic operators" -- e.g. *very*, *more*, *more* or *less*, *much*, *essentially*, *slightly* etc., which he called "hedges".]

Zadeh L.A. (1973). Outline of a new approach to the analysis of complex systems and decision processes, *IEEE Trans. on Systems, Man, and Cybernetics* **SMC-3**(1), 28--44. [In this article Zadeh treated fuzzy algorithms and he also integrated the other fuzzifications into a new approach that was supposed to bring about a completely new form of system analysis based on his Fuzzy Set Theory by using Linguistic Variables, Fuzzy If-Then-rules, and Fuzzy Algorithms.]

Zadeh L.A. (1975). Fuzzy Logic and Approximate Reasoning, *Synthese* **30**, 407--428. [Zadeh's article on the imprecise logical system, FL, in a philosophical journal. In FL the truth-values are fuzzy sets of the unit interval with linguistic labels such as *true*, *false* but also *not true*, *very true*, *quite true*, *not very true* and *not very false*, etc.]

Zadeh L.A. (1975). The Concept of a Linguistic Variable and its Application to Approximate Reasoning -- I, *Information Sciences* **8**, 199--249; -- II, *Inf. Sci.* **8**, 301--357; -- III, *Inf. Sci.* **9**, 43—80. [In this series of three articles Zadeh introduces his linguistic approach to Approximate Reasoning: By *linguistic variables* as variables whose values are words or sentences in a natural or artificial language. He expected that the main applications of the linguistic approach lie in the realm of humanistic systems, particularly in the fields of artificial intelligence, linguistics, human decision processes, pattern recognition, psychology, law, medical diagnosis, information retrieval, economics and related areas.]

Zadeh L.A. (1978). PRUF – a meaning representation language for natural languages, *Int. J. of Man-Machine Studies* **10**, 395—460. [In this article Zadeh describes the relation-manipulating language *Probabilistic Relational Universal Fuzzy* (PRUF), to precisiate expressions in a natural language; to exhibit their logical structure; and to provide a system for the characterization of the meaning of a proposition by acting on a collection of fuzzy relations in a data base and returning a possibility distribution.]

Zadeh L.A. (1994). Interview with Lotfi Zadeh, Creator of Fuzzy Logic by Betty Blair, *Azerbaijada International* **2**(4), winter. [An interesting interview on the history and the future of Fuzzy Sets and Systems with the founder of this theory.]

Zadeh L.A. (1996). Fuzzy Logic = Computing with Words, *IEEE Transactions on Fuzzy Systems* **4**(2), 103-111. [Computing with Words (CW) is introduced in this article as a methodology in which words are used in place of numbers for computing and reasoning. In CW, a word is viewed as a label of a fuzzy set of points drawn together by similarity, with the fuzzy set playing the role of a fuzzy constraint on a variable.]

Zadeh L.A. (1999). From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions, *IEEE Trans. on Circuits And Systems-I: Fundamental Theory and Applications* **45**(1), 105-119. [In this article Zadeh proposed Computing with Words (CW) based on the theories of Fuzzy Sets and Systems and Fuzzy Logic and these methodologies instead of exact Computing with numbers. In this article he wrote that "the main contribution of fuzzy logic is a methodology for computing with words. No other methodology serves this purpose."]

Zadeh L.A. (1999). The Birth and Evolution of Fuzzy Logic – A Personal Perspective, *Journal of Japan Society for Fuzzy Theory and Systems* **11**(6), 891-905. [In this article Zadeh gives a brief and personal sketch of the genesis and development of Fuzzy Sets and Systems and Fuzzy Logic and he mentions positive and negative reactions to his theory.]

Zadeh L.A. (2001). New Direction in AI. Toward a Computational Theory of Perceptions, *AI Magazine* **22**(1), 73-84. [In this article Zadeh outlines the Computational Theory of Perceptions (CTP). CTP as a new direction in AI is intended to be a capability to compute and reason with perception-based information. Perceptions would be described by propositions drawn from a natural language.]

Zadeh L.A. 1984). Making Computers Think like People, *IEEE Spectrum* **8**, 26-32. [In this article Zadeh focused on the machine's ability to "compute with numbers" that he intends to supplement by an additional ability that is similar to human thinking. The "remarkable human capability [of humans] to perform a wide variety of physical and mental tasks without any measurements and any computations."]

Zadeh L.A. and Polak E. (1969) *System Theory*, Bombay, New Delhi: McGraw-Hill. [This vol. 8 in the Inter-University electronics series is a collection of papers on system theory.]

Zadeh L.A., Desoer C. A. (1963). *Linear System Theory: The State Space Approach*. New York: McGraw-Hill Book Company. [This textbook for engineers in research and development and applied mathematicians is landmark in the development of the state space approach. It concerns the technique's application to systems described by differential equations.]

Zadeh L.A., Miller K.S. (1952) Generalized Ideal Filters, *Journal of Applied Physics* **23**(2), 223--228. [In this paper the authors established their general theory of linear signal transmission systems with extensive use of modern mathematical methods: Fourier analysis as well as Hilbert space and operator calculus]

Biographical Sketch

Rudolf Seising is an Adjoint Researcher at the European Centre for Soft Computing in Mieres (Asturias), Spain. His main areas of research comprise historical and philosophical foundations of science and technology.

After a study of Mathematics and Physics at the Ruhr-University of Bochum (Germany) he obtained his Ph.D. at the Faculty of Philosophy, Philosophy of Science, and Statistics of the Ludwig–Maximilians–University (LMU) in Munich (Germany), with a thesis on "Probabilistic Structures in Quantum Mechanics". At the same university he completed later the German Habilitation in history of science with the thesis: "The Fuzzification of Systems: The Genesis of the Theory of Fuzzy Sets and their first Applications – Their Development until the 70s in the 20th century". (Springer 2007).

Among other books he edited "Views on Fuzzy Sets and Systems from Different Perspectives. Philosophy and Logic, Criticisims and Applications". (Springer-Verlag 2009), (together with Veroníca

Sanz) “Soft Computing in Humanities and Social Sciences” (Springer 2012), (together with E. Trillas, S. Termini, and C. Moraga two volumes of “On Fuzziness – A Homage to Lotfi A. Zadeh” (Springer 2013).

Dr. Seising has been Scientific Assistant for computer sciences at the University of the Armed Forces in Munich from 1988 to 1995 and for history of sciences at the same university from 1995 to 2002. From 2002 to 2008 he was with the Core unit for Medical Statistics and Informatics at the University of Vienna Medical School, which in 2004 became the Medical University of Vienna. Since 2005 he is College Lecturer at the Faculty of History and Arts, at the LMU in Munich. In 2008 he was acting as Professor for the history of science at the Friedrich-Schiller-University Jena (Germany) and in 2009/10 at the LMU in Munich. He has been several times Visiting Scholar at the University of California, Berkeley.

Since 2004 he is Chairman of the IFSA Special Interest Group “History” and since 2007 of the EUSFLAT Working Group “Philosophical Foundations”.